Theory of Time-Domain Quasi-TEM Modes in Inhomogeneous Multiconductor Lines

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Abstract—Theory for quasi-TEM modes propagating in a transversely inhomogeneous (multidielectric), longitudinally uniform transmission line, previously derived for time-harmonic waves, is derived for transient signals. It is seen that, while the starting point for the theory is completely different, the result is similar to the time-harmonic theory, and previously derived properties for propagating modes also apply in the transient case. The range of applicability is discussed with a simple example.

I. Introduction

THE INHOMOGENEOUS multiconductor transmission line is interesting because of the wide field of applications of the microstrip structure. In the case of time-harmonic (sinusoidal) fields, the exact TEM modes for this kind of geometry, do not exist. Before the introduction of a theory for the quasi-TEM modes, there were often erroneous assumptions about the nature of fields propagating in such structures. The exact theory for two-conductor lines was first given by dos Santos and Figanier in 1975 [1], generalized to multiconductor lines by Mannersalo in 1977 (not published) and, in a more complete form, by Lindell in 1981 [2]. There also exist other expositions on the subject [3], [4].

As pointed out in [2], the transverse quasi-TEM mode field is a kind of glued-together pair of static electric and static magnetic fields, whose boundary conditions are connected through a pair of consistency equations, which also determine the propagation properties of the quasi-TEM wave. The longitudinal components can be calculated from the transverse components and they are low for small frequencies. This theory was obtained by expanding all unknown quantities in series of ascending powers of the frequency. How this can be generalized to fields of arbitrary time dependence is not evident, because there does not exist a small parameter like ω. However, obviously, a similar theory should apply for transient signals whose spectrum consists of sufficiently low frequencies. A need for a firm foundation for such a theory exists because of the new generation of computers applying microstriplike geometry and rapid pulsed signals, although the important wavelengths of such signals are still large with respect to transverse measures of the line. Calculations of coupling between such signals applying the ideal TEM mode approximation have recently been presented [5], [6].

The theory given here is not based on an asymptotic (perturbational) series approach, but on an iterational consideration, which starts on a quasi-TEM assumption, proceeds to derive properties for such a field, and finally tries to find out conditions of validity for the original assumption.

II. The Iteration Method

The following theory of mode propagation in a transversely inhomogeneous multiconductor waveguide, Fig. 1, is a companion to that given in [2] and uses much of the same notation, the main difference being that the subscripts z are deleted. The electromagnetic field is studied in terms of its transverse and longitudinal components, respectively, transverse and parallel to the guide axis vector $\bar{u}$:

$$\bar{E}(\bar{r},t)=\bar{e}(\bar{r},t)+\bar{u}e(\bar{r},t) \tag{1}$$

$$\bar{H}(\bar{r},t)=\bar{h}(\bar{r},t)+\bar{u}h(\bar{r},t). \tag{2}$$

When the Maxwell equations are written for the decomposed fields and the resulting equations are decomposed into axial and transverse components, we have the set of equations

$$\nabla_\perp\times\bar{e}=-\bar{u}\mu\partial_t h \tag{3}$$

$$\nabla_\perp\times\bar{h}=\bar{u}\epsilon\partial_t e \tag{4}$$

$$\nabla_\perp e=-\partial_z\bar{e}+\partial_t\bar{u}\times(\mu\bar{h}) \tag{5}$$

$$\nabla_\perp h=-\partial_z\bar{h}-\partial_t\bar{u}\times(\epsilon\bar{e}) \tag{6}$$

$$\nabla_\perp\cdot(\epsilon\bar{e})=-\epsilon\partial_z e \tag{7}$$

$$\nabla_\perp\cdot(\mu\bar{h})=-\mu\partial_z h. \tag{8}$$

Here, $\nabla_\perp$ denotes the transverse component of ∇; μ and ϵ are functions of the transverse vector $\bar{\rho}$, and the partial derivatives $\partial F/\partial\xi$ are denoted in short by $\partial_\xi F$.

Equations (3)–(8) are exact. As a starting point to the iteration method leading to the quasi-TEM theory, we assume that the right-hand sides of (3), (4), (7), and (8) are small and approximate them by zeros. This assumption will be studied more closely later on. After solving for the transverse fields on the left-hand side, the longitudinal

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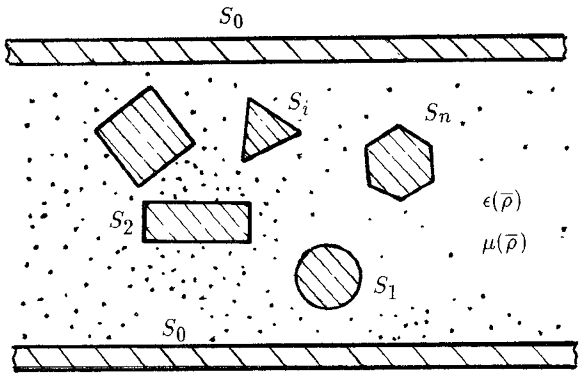

Fig 1. Cross section of the transversely inhomogeneous multiconductor line with N conductors S_i and a conducting sheath S_0. The unit vector $\bar{u}$ and z coordinate are perpendicular to the plane.

fields are solved from (5) and (6). The iteration could be continued by substituting these on the right-hand sides of (3), (4), (7), and (8), and solving for the next approximations for the transverse fields. The process will evidently converge if the longitudinal fields are small enough. Here, we are only concerned with the first round of iteration, which gives us static approximation of the transverse fields and quasi-static approximation of longitudinal fields.

III. Static Approximation

Setting $e = 0$ and $h = 0$ in the four equations (3), (4), (7), and (8) gives us equations for the first approximation of the transverse fields $\bar{e}_0, \bar{h}_0$:

$$\nabla_\perp \times \bar{e}_0(\bar{r}, t) = 0 \tag{9}$$

$$\nabla_\perp \cdot (\epsilon \bar{e}_0) = 0 \tag{10}$$

$$\nabla_\perp \times \bar{h}_0(\bar{r}, t) = 0 \tag{11}$$

$$\nabla_\perp \cdot (\mu \bar{h}_0) = 0. \tag{12}$$

Equations (9) and (10) constitute an electrostatic problem in two dimensions, and as the boundary conditions on the conductors we have

$$\bar{n} \times \bar{e}_0 = 0. \tag{13}$$

Correspondingly, (11) and (12) define a magnetostatic problem with the boundary conditions

$$\bar{n} \cdot \bar{h}_0 = 0. \tag{14}$$

Here, $\bar{n}$ denotes the unit vector normal to the conductor boundaries. The static problems can be treated with potential quantities. In fact, defining the electric field in terms of a scalar potential ϕ:

$$\bar{e}_0(\bar{r}, t) = -\nabla_\perp \phi(\bar{r}, t) \tag{15}$$

(9) will be automatically satisfied, whence the equations for ϕ, from (10) and (13), are

$$\nabla_\perp \cdot (\epsilon \nabla_\perp \phi) = 0 \tag{16}$$

$$\phi(\bar{r}, t) = U_i(z, t) \qquad \text{for } \bar{\rho} \in S_i. \tag{17}$$

Here, S_i denotes the surface of the ith conductor, $i = 0, 1, \cdots, N$. For simplicity, we assume that there is a sheath conductor denoted by $i = 0$ and define $U_0 = 0$ so that U_i means the voltage between the ith conductor and the sheath. The present theory is not limited to closed geometries. If the correct behavior of potential functions in the infinity is taken into account, the analysis can be written in very much the same fashion. However, instead of defining voltages with respect to the sheath, another reference potential ("ground") must be assumed.

The solution of (16) and (17) obviously depends on the set of boundary values $U_i(z, t)$, which can be represented as an N-vector $\boldsymbol{U}(z, t)$. To be able to write the solution for any boundary values, we must solve N different normalized problems for functions $\phi_i(\bar{\rho})$, making up an N-vector function $\boldsymbol{\phi}(\bar{\rho})$, satisfying the boundary conditions $\phi_j = 1$ on S_j and $\phi_j = 0$ on all the other conductors $(i \neq j)$. The solution corresponding to the boundary value vector $\boldsymbol{U}(z, t)$ can then be written as

$$\phi(\bar{r}, t) = \sum_{i=1}^{N} U_i(z, t) \phi_i(\bar{\rho}) = \boldsymbol{U}(z, t) \circ \boldsymbol{\phi}(\bar{\rho}). \tag{18}$$

Here, we have adopted the notation $\circ$ for the inner product of two N-vectors to distinguish it from the "dot" product of two vectors in the physical three-space.

In the same manner, the magnetostatic problem can be formulated in terms of an axial vector potential $\bar{A} = \bar{u}A$:

$$\mu(\bar{\rho}) \bar{h}_0(\bar{r}, t) = \nabla_\perp A(\bar{r}, t) \times \bar{u}. \tag{19}$$

Equation (12) is automatically satisfied through (19). Out of (11) and (14) comes

$$\nabla_\perp \cdot \left(\frac{1}{\mu} \nabla_\perp A \right) = 0 \tag{20}$$

$$A(\bar{r}, t) = \Psi_i(z, t) \qquad \text{for } \bar{\rho} \in S_i. \tag{21}$$

Here, Ψ_i denotes the magnetic flux/unit length between the conductor i and the sheath. In terms of a general solution N-vector $\boldsymbol{A}(\bar{\rho})$ and a set of boundary values $\boldsymbol{\Psi}(z, t)$ the solution can be written as

$$A(\bar{r}, t) = \boldsymbol{\Psi}(z, t) \circ \boldsymbol{A}(\bar{\rho}). \tag{22}$$

In the electrostatic problem (16), (17), the boundary-value voltage N-vector uniquely defines the quantities Q_i, i.e., the charge/unit length on each conductor through

$$Q_i(z, t) = \oint_{C_i} \epsilon \bar{e}_0 \cdot \bar{n} \, dl = -\oint_{C_i} \epsilon \frac{\partial \phi}{\partial n} \, dl \tag{23}$$

where C_i denotes the circumference of the ith conductor. The linear relation between the voltage and charge N-vectors can be written in terms of a static capacitance/unit length matrix $\underline{\underline{C}}$

$$\boldsymbol{Q}(z, t) = \underline{\underline{C}} \circ \boldsymbol{U}(z, t). \tag{24}$$

For the magnetostatic problem, there is also a linear relation between the magnetic fluxes and the currents I_i on the conductors, defined by

$$I_i(z, t) = \oint_{C_i} \bar{h}_0 \cdot d\bar{l} = -\oint_{C_i} \frac{1}{\mu} \frac{\partial A}{\partial n} \, dl \tag{25}$$

$$\boldsymbol{\Psi}(z, t) = \underline{\underline{L}} \circ \boldsymbol{I}(z, t) \tag{26}$$

where $\underline{\underline{L}}$ is the inductance/unit length matrix.

IV. The Quasi-TEM Field

Until now, the two static problems have no connection. The boundary value N-vectors are, however, coupled through (5) and (6). From (5), we can solve the approximate longitudinal component e_1:

$$e_1(\bar{r},t) = \partial_z \phi(\bar{r},t) + \partial_t A(\bar{r},t). \tag{27}$$

The corresponding approximation for the longitudinal magnetic field h_1 cannot be obtained from (6) explicitly:

$$\nabla_\perp h_1(\bar{r},t) = -\partial_z \bar{h}_0(\bar{r},t) - \bar{u}\times\epsilon\partial_t \bar{e}_0(\bar{r},t)$$
$$= \bar{u}\times\left[\frac{1}{\mu}\nabla_\perp \partial_z A(\bar{r},t) + \epsilon\nabla_\perp \partial_t \phi(\bar{r},t)\right]. \tag{28}$$

Because the right-hand side of (28) is curl free, as can easily be checked, h_1 can be obtained through integration from a reference point $\bar{\rho}_0$ to the general point $\bar{\rho}$. Integrating round the conductor i, the integral should vanish since h_1 is a unique physical quantity. To obtain unique values for h_1 from (28) by integration, one more condition for h_1 is needed, because the reference value $h_1(\bar{\rho}_0)$ is not known. The missing condition is obtained by integrating (3) over the waveguide cross section, which gives us zero if Stokes' theorem is invoked, because the line integral of the electric field around the perimeter vanishes. Thus, $\partial_t \int h dS = 0$, or the integral is constant in time. If the fields are switched on at some finite time, the constant is zero and we have for the additional condition for h_1

$$\int_S h_1(\bar{r},t)\, dS = 0 \tag{29}$$

where S is the total cross section surface of the waveguide.

Because of the boundary condition $e_1 = 0$ on the conductors, (27) defines a relation between the boundary values of the potentials ϕ and A:

$$\partial_z \boldsymbol{U}(z,t) + \partial_t \boldsymbol{\Psi}(z,t) = 0. \tag{30}$$

This is, in fact, one form of writing the Faraday law, in which a moving magnetic flux produces an induced voltage. The other equation, (28), gives rise to another relation between integrated quantities of the potentials ϕ and A, in the form of a continuity equation:

$$\partial_z \boldsymbol{I}(z,t) + \partial_t \boldsymbol{Q}(z,t) = 0. \tag{31}$$

Substituting from (24) and (26), we have

$$\partial_z \boldsymbol{U}(z,t) = -\underline{\underline{L}}\circ\partial_t \boldsymbol{I}(z,t) \tag{32}$$

$$\partial_z \boldsymbol{I}(z,t) = -\underline{\underline{C}}\circ\partial_t \boldsymbol{U}(z,t). \tag{33}$$

These are recognized as transmission-line equations for a multiconductor line. Eliminating $\boldsymbol{I}$, we have the wave equation

$$\left(\partial_z^2 \underline{\underline{I}} - \underline{\underline{L}}\circ\underline{\underline{C}}\partial_t^2\right)\circ\boldsymbol{U}(z,t) = 0. \tag{34}$$

Here, $\underline{\underline{I}}$ denotes the $N\times N$ unit matrix. Limiting ourselves to the solution traveling in the positive z direction, the operator in (34) can be halved to produce the equation

$$\left(\partial_z \underline{\underline{I}} + \left(\underline{\underline{L}}\circ\underline{\underline{C}}\right)^{1/2}\partial_t\right)\circ\boldsymbol{U}(z,t) = 0. \tag{35}$$

Because $\underline{\underline{C}}$ and $\underline{\underline{L}}$ are positive definite and symmetric matrices, there exists a square-root matrix $(\underline{\underline{L}}\circ\underline{\underline{C}})^{1/2}$ which is positive definite. Hence, it possesses a complete set of eigenvectors and positive eigenvalues, and the solution of (35) can be written in terms of the solutions of

$$\left(\underline{\underline{L}}\circ\underline{\underline{C}}\right)^{1/2}\circ\boldsymbol{V}^j = \frac{1}{v_j}\boldsymbol{V}^j. \tag{36}$$

Corresponding to each eigenvector $\boldsymbol{V}^j$ there exists a mode of the original problem, whose voltage vector can be written as

$$\boldsymbol{U}^j(z,t) = \boldsymbol{V}^j f_j(z - v_j t) \tag{37}$$

with an arbitrary function f_j. Thus, the eigenvalues of the algebraic equation (36) define the velocities of the N quasi-TEM modes on the inhomogeneous waveguide. The current distribution vector corresponding to the jth mode, $\boldsymbol{I}^j$, satisfies similar equations but with $\underline{\underline{L}}$ and $\underline{\underline{C}}$ interchanged. As in [2], this leads to an eigenvector different from $\boldsymbol{U}^j$, in general, which means that there does not exist a simple scalar impedance, but the impedance is a matrix quantity, denoted here by $\underline{\underline{Z}}$.

V. Traveling Wave Solutions

The practical multiconductor line problem, with possible small deviation from axial uniformity, can be treated with traveling wave quantities with less effort than with current/voltage quantities. In fact, we can define the voltage wave vectors [7] by

$$\boldsymbol{U}_\pm = \boldsymbol{U} \pm \underline{\underline{Z}}\circ\boldsymbol{I} \tag{38}$$

where the impedance matrix is defined in any of the following equivalent forms [2]:

$$\underline{\underline{Z}} = \left(\underline{\underline{L}}\circ\underline{\underline{C}}\right)^{1/2}\circ\underline{\underline{C}}^{-1} = \underline{\underline{C}}^{-1}\circ\left(\underline{\underline{C}}\circ\underline{\underline{L}}\right)^{1/2}$$
$$= \left(\underline{\underline{L}}\circ\underline{\underline{C}}\right)^{-1/2}\circ\underline{\underline{L}} = \underline{\underline{L}}\circ\left(\underline{\underline{C}}\circ\underline{\underline{L}}\right)^{-1/2}. \tag{39}$$

It must be noted that since in the general case the matrices $\underline{\underline{C}}$ and $\underline{\underline{L}}$ do not commute, the square root of their product is dependent on the order of the product. Solving the voltage and current vectors from (38) in terms of voltage waves and substituting in (32) give us the two equations

$$\partial_z \boldsymbol{U}_\pm(z,t) \pm \left(\underline{\underline{L}}\circ\underline{\underline{C}}\right)^{1/2}\circ\partial_t \boldsymbol{U}_\pm(z,t) = 0. \tag{40}$$

It is seen that equations (40) are uncoupled for both traveling voltage vectors. Thus, they possess solutions of the type

$$\boldsymbol{U}^j_\pm(z,t) = \boldsymbol{V}^j f_j(z \mp v_j t). \tag{41}$$

For a line with slight nonuniformity along its axis, equations (40) can be generalized to possess a perturbational coupling term on the right-hand side [5], [6].

It is also easy to show that the integral of the Poynting vector $\frac{1}{2}\bar{e}_0\times\bar{h}_0^*$ over the cross section equals the sum of $\frac{1}{2}\boldsymbol{U}\circ\boldsymbol{I}^*$, or the propagating power in the quasi-TEM mode can be expressed in terms of fields or boundary values.

VI. On the Validity of the Quasi-TEM Concept

After having calculated the approximate longitudinal field components in terms of the approximate transverse components, we can substitute them on the right-hand sides of (3), (4), (7), and (8) and calculate a better approximation for the transverse fields, $\bar{e}_2, \bar{h}_2$. For these fields not to differ from the original approximations $\bar{e}_0, \bar{h}_0$ significantly, the right-hand sides should be small. What does this mean? Obviously, in order to be able to approximate the transverse curl operations in (3) and (4) by zero, the right-hand sides should be small with respect to other derivatives than curl of the vector functions on the left-hand sides. In this case, the iteration would converge and the terms already calculated would present a reasonable approximation to the fields.

Let us study the question in terms of simple consideration of order. If, for example, the following inequality is valid:

$$\|\nabla_\perp \bar{e}_0\| \gg |\mu \partial_t h_1| \tag{42}$$

then obviously from (3), $\nabla \times \bar{e}_0$ is small when compared to other transverse derivatives of $\bar{e}_0$ and it can be approximated by zero. Thus, when solving the next iterational step, the transverse field $\bar{e}_2$ in terms of h_1 from (3), the solution $\bar{e}_2$ does not differ from $\bar{e}_0$ very much. Now the order of $\nabla_\perp$ is $1/D$, where D is the transverse dimension of the guide. If the dimension of the axial variation of the fields is L, we have $f'(z-v_p t)$ of the order $f(z-v_p t)/L$, where v_p is the phase velocity of the mode and from (28) and (42) we may write

$$|\bar{e}_0| \gg \mu v_p \left(\frac{D}{L}\right)^2 |-\bar{h}_0 + v_p \bar{u} \times \epsilon \bar{e}_0|. \tag{43}$$

This inequality is obviously valid if D/L is small enough, i.e., if the transverse dimension of the guide is small enough with respect to field variations in the z direction. In other words, the variation of the signal should be so slow that the propagating field does not change much at the distance of the transverse dimension in z direction. But this is not all, because the vector term on the right-hand side may also be small. This happens if the inhomogeneity of the guide is small. In fact, if the guide becomes homogeneous, the solution of (20) becomes a multiple of the solution of (16) and the relation obtained from (30) is $\phi = v_p A$, or from (15) and (19) $\bar{h}_0 = \bar{u} \times \bar{e}_0/\eta$, with $\eta = \mu v_p = 1/\epsilon v_p$, which makes the right-hand side of (43) vanish. If the inhomogeneity is small, the term need not vanish, but it is small and (43) is valid for larger values of the ratio D/L.

There does not seem to be an easy way to express the condition of validity in more exact terms. A simpler condition is obtained if the ratios $|e_1|/|\bar{e}_0|, |h_1|/\bar{h}_0|$ are considered. This not only shows us that the basic approximation of the field is quasi TEM, but also gives an idea on the convergence. In fact, writing

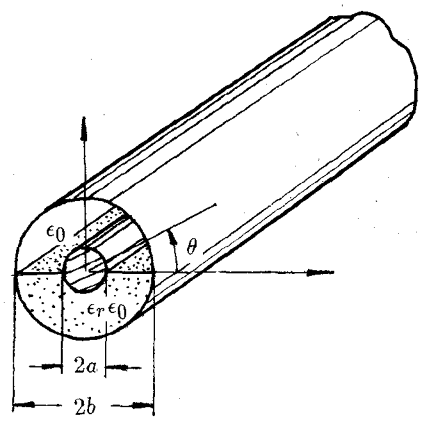

Fig. 2. The inhomogeneous coaxial cable. Basic wave is a TE wave, which is quasi-TEM for sufficiently slowly varying transient signals and/or small inhomogeneity (small $\epsilon_r - 1$).

$$\frac{|e_1|}{|\bar{e}_0|} = \left|\frac{f'(z-v_p t)}{f(z-v_p t)}\right| \frac{|\phi - v_p A|}{|\nabla_\perp \phi|} = \left|\frac{f'}{f}\right| \frac{|(\phi - A) \circ V|}{|\nabla_\perp \phi \circ V|} \tag{44}$$

we may require this to satisfy the condition $\ll 1$. The last factor is zero for homogeneous line and small for small inhomogeneity. Because the maximum values of ϕ and A are their boundary value 1, it is obviously bounded by the value $2D$, which shows us that the quasi-TEM mode is always possible if the signal is varying slowly enough.

VII. Example

Let us elucidate the problem of convergence through a simple example of inhomogeneous coaxial cable with $\mu = \mu_0$ and $\epsilon_r = 1$ for $\theta = \theta \cdots \pi$ and $\epsilon_r \neq 1$ for $\theta = 0 \cdots -\pi$, Fig. 2. Because of the special symmetry, the potential fields ϕ and A are multiples of the same function $g(\rho)$:

$$g(\rho) = \frac{\ln\left(\frac{b}{\rho}\right)}{\ln\left(\frac{b}{a}\right)}. \tag{45}$$

The inductance is the same as for the homogeneous coaxial line:

$$L = L_0 = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \tag{46}$$

and the capacitance equals

$$C = \left(\frac{1+\epsilon_r}{2}\right) C_0 = \frac{\pi(1+\epsilon_r)\epsilon_0}{\ln\left(\frac{b}{a}\right)}. \tag{47}$$

The phase velocity of the quasi-TEM wave is

$$v_p = \frac{1}{\sqrt{L_0 C}} = c\sqrt{\frac{2}{1+\epsilon_r}}. \tag{48}$$

The potentials are

$$\phi(\bar{r},t) = Vg(\rho)f(z - v_p t) \quad (49)$$

$$A(\bar{r},t) = \Psi g(\rho)f(z - v_p t) = \frac{V}{v_p} g(\rho) f(z - v_p t). \quad (50)$$

From (27) we have

$$e_1 = Vg(\rho)f'(z - v_p t) - v_p \frac{V}{v_p} g(\rho) f'(z - v_p t) = 0 \quad (51)$$

or the longitudinal electric field vanishes in this approximation. Thus, the quasi-TEM is in fact a TE field in this approximation, which is due to the special symmetry of the structure. From (28) we have

$$h_1(\bar{r},t) = \frac{V}{\eta} f'(z - v_p t) \int_{\bar{r}_0}^{\bar{r}} \left(\frac{c}{v_p} - \epsilon_r(\bar{\rho}) \frac{v_p}{c} \right) \cdot (\bar{m} \cdot \nabla_\perp g)\, dC + h_1(\bar{r}_0). \quad (52)$$

The unit vector $\bar{m}$ is normal to the integration path. Because $\bar{m} \cdot \nabla_\perp g(\rho) = 0$ on each radial line, it is clear that h_1 is a function of θ only. Thus, the integration path can be taken along any circle of radius ρ. If the constant $h_1(\bar{r}_0)$ is determined through the condition (29), the result is

$$h_1(\bar{r},t) = \frac{V}{\eta \ln(b/a)} f'(z - v_p t) \frac{\epsilon_r - 1}{\sqrt{2(\epsilon_r + 1)}} \left(|\theta| - \frac{\pi}{2} \right) \quad \text{for } -\pi \leq \theta \leq \pi. \quad (53)$$

This expression is seen to vanish for $\epsilon_r = 1$, in which case the guide is homogeneous. The null field is obtained for $\theta = \pm \pi/2$ and maxima at $\theta = 0, \pi$. To check the quasi-TEM character of the wave, we compare the magnitudes of h_1 and $\bar{h}_0$:

$$\frac{|h_1|}{|\bar{h}_0|} = \left(\frac{\epsilon_r - 1}{\epsilon_r + 1} \right) \left| \frac{f'(z - v_p t)}{f(z - v_p t)} \right| \left| \left(|\theta| - \frac{\pi}{2} \right) \rho \right|. \quad (54)$$

The maximum value of the last factor is $\pi b/2$. Thus, the right-hand side of (54) gives us the relative rate of change of the signal in the axial direction over the distance $(\epsilon_r - 1)\pi b / 2(\epsilon_r + 1)$. Obviously, the convergence of the iteration is good and the quasi-TEM concept valid if this rate is small.

VIII. Conclusions

The quasi-TEM mode theory of inhomogeneous multiconductor waveguides, previously presented for time-harmonic fields, was generalized for fields with arbitrary time dependence. The theory is based upon an iterative approach, and the condition for its convergence was outlined and elucidated with a simple example. It was seen that the resulting theory, basically similar to that given for the time-harmonic case, can be applied for transient signals, provided the variation of the signal is slow enough or the inhomogeneity is not too large.

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References

[1] A. F. dos Santos, and J. P. Figanier, "The method of series expansion in the frequency domain applied to multidielectric transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 753–756, Sept. 1975.

[2] I. V. Lindell, "On the quasi-TEM modes in inhomogeneous multiconductor transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 812–817, Aug. 1981.

[3] M. Aubourg, "Onde quasi-TEM des guides cylindriques blindés," *Annales de Télécommunication*, vol. 34, nos. 1–2, pp. 45–51, 1979.

[4] C. Vassallo, *Théorie des Guides d'Ondes Électromagnétiques*. Paris: Eyrolles, 1985.

[5] Y-C. E. Yang, J. A. Kong, and Q. Gu, "Time-domain perturbational analysis of nonuniformly coupled transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1120–1130, Nov. 1985.

[6] Q. Gu, J. A. Kong, and Y-C. E. Yang, "Time domain analysis of nonuniformly coupled line systems," *JEWA*, vol. 1, no. 2, pp. 109–132, 1987.

[7] P. E. Adair and G. I. Haddad, "Coupled-mode analysis of nonuniform coupled transmission lines," *IEEE Trans. on Microwave Theory Tech.*, vol. MTT-17, pp. 746–752, Oct. 1969.

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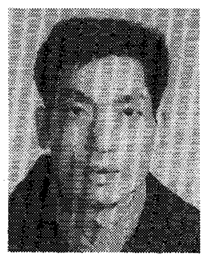

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